Determine the modulus of resilience for each of the following alloys:

<table>
<thead>
<tr>
<th>Material</th>
<th>MPa</th>
<th>psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel alloy</td>
<td>550</td>
<td>80,000</td>
</tr>
<tr>
<td>Brass alloy</td>
<td>350</td>
<td>50,750</td>
</tr>
<tr>
<td>Aluminum alloy</td>
<td>250</td>
<td>36,250</td>
</tr>
<tr>
<td>Titanium alloy</td>
<td>800</td>
<td>116,000</td>
</tr>
</tbody>
</table>

Use modulus of elasticity values in Table 6.1.

**Solution**

The moduli of resilience of the alloys listed in the table may be determined using Equation 6.14. Yield strength values are provided in this table, whereas the elastic moduli are tabulated in Table 6.1.

For steel

\[
U_r = \frac{\sigma_y^2}{2E}
\]

\[
U_r = \frac{(550 \times 10^6 \text{ N/m}^2)^2}{(2)(207 \times 10^9 \text{ N/m}^2)} = 7.31 \times 10^5 \text{ J/m}^3 \quad (107 \text{ in.-lbf/in.}^3)
\]

For the brass

\[
U_r = \frac{(350 \times 10^6 \text{ N/m}^2)^2}{(2)(97 \times 10^9 \text{ N/m}^2)} = 6.31 \times 10^5 \text{ J/m}^3 \quad (92.0 \text{ in.-lbf/in.}^3)
\]

For the aluminum alloy

\[
U_r = \frac{(250 \times 10^6 \text{ N/m}^2)^2}{(2)(69 \times 10^9 \text{ N/m}^2)} = 4.53 \times 10^5 \text{ J/m}^3 \quad (65.7 \text{ in.-lbf/in.}^3)
\]

And, for the titanium alloy

\[
U_r = \frac{(800 \times 10^6 \text{ N/m}^2)^2}{(2)(107 \times 10^9 \text{ N/m}^2)} = 30.0 \times 10^5 \text{ J/m}^3 \quad (434 \text{ in.-lbf/in.}^3)
\]
A tensile test is performed on a metal specimen, and it is found that a true plastic strain of 0.20 is produced when a true stress of 575 MPa (83,500 psi) is applied; for the same metal, the value of $K$ in Equation 6.19 is 860 MPa (125,000 psi). Calculate the true strain that results from the application of a true stress of 600 MPa (87,000 psi).

**Solution**

It first becomes necessary to solve for $n$ in Equation 6.19. Taking logarithms of this expression and after rearrangement we have

$$n = \frac{\log \sigma_T - \log K}{\log \varepsilon_T}$$

And, incorporating values of the parameters provided in the problem statement leads to

$$n = \frac{\log (575 \text{ MPa}) - \log (860 \text{ MPa})}{\log (0.20)} = 0.250$$

Expressing $\varepsilon_T$ as the dependent variable (Equation 6.19), and then solving for its value from the data stipulated in the problem statement, leads to

$$\varepsilon_T = \left( \frac{\sigma_T}{K} \right)^{1/n} = \left( \frac{600 \text{ MPa}}{860 \text{ MPa}} \right)^{1/0.250} = 0.237$$
6.45 For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strains, prior to necking:

<table>
<thead>
<tr>
<th>Engineering Stress (MPa)</th>
<th>Engineering Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>235</td>
<td>0.194</td>
</tr>
<tr>
<td>250</td>
<td>0.296</td>
</tr>
</tbody>
</table>

On the basis of this information, compute the engineering stress necessary to produce an engineering strain of 0.25.

**Solution**

For this problem we first need to convert engineering stresses and strains to true stresses and strains so that the constants $K$ and $n$ in Equation 6.19 may be determined. Since $\sigma_T = \sigma (1 + \varepsilon)$ then

$$\sigma_{T1} = (235 \text{ MPa})(1 + 0.194) = 280 \text{ MPa}$$

$$\sigma_{T2} = (250 \text{ MPa})(1 + 0.296) = 324 \text{ MPa}$$

Similarly for strains, since $\varepsilon_T = \ln (1 + \varepsilon)$ then

$$\varepsilon_{T1} = \ln (1 + 0.194) = 0.177$$

$$\varepsilon_{T2} = \ln (1 + 0.296) = 0.259$$

Taking logarithms of Equation 6.19, we get

$$\log \sigma_T = \log K + n \log \varepsilon_T$$

which allows us to set up two simultaneous equations for the above pairs of true stresses and true strains, with $K$ and $n$ as unknowns. Thus

$$\log (280) = \log K + n \log (0.177)$$

$$\log (324) = \log K + n \log (0.259)$$
Solving for these two expressions yields \( K = 543 \text{ MPa} \) and \( n = 0.383 \).

Now, converting \( \varepsilon = 0.25 \) to true strain

\[
\varepsilon_T = \ln (1 + 0.25) = 0.223
\]

The corresponding \( \sigma_T \) to give this value of \( \varepsilon_T \) (using Equation 6.19) is just

\[
\sigma_T = K \varepsilon_T^n = (543 \text{ MPa})(0.223)^{0.383} = 306 \text{ MPa}
\]

Now converting this value of \( \sigma_T \) to an engineering stress using Equation 6.18a gives

\[
\sigma = \frac{\sigma_T}{1 + \varepsilon} = \frac{306 \text{ MPa}}{1 + 0.25} = 245 \text{ MPa}
\]
6.51  (a) A 10-mm-diameter Brinell hardness indenter produced an indentation 1.62 mm in diameter in a steel alloy when a load of 500 kg was used. Compute the HB of this material.

(b) What will be the diameter of an indentation to yield a hardness of 450 HB when a 500 kg load is used?

Solution

(a) We are asked to compute the Brinell hardness for the given indentation. It is necessary to use the equation in Table 6.5 for HB, where $P = 500$ kg, $d = 1.62$ mm, and $D = 10$ mm. Thus, the Brinell hardness is computed as

$$\text{HB} = \frac{2P}{\pi D \left[ D - \sqrt{D^2 - d^2} \right]}$$

$$= \frac{(2)(500 \text{ kg})}{(\pi)(10 \text{ mm}) \left[ 10 \text{ mm} - \sqrt{(10 \text{ mm})^2 - (1.62 \text{ mm})^2} \right]} = 241$$

(b) This part of the problem calls for us to determine the indentation diameter $d$ which will yield a 450 HB when $P = 500$ kg. Solving for $d$ from the equation in Table 6.5 gives

$$d = \sqrt{D^2 - \left[ D - \frac{2P}{(\text{HB})\pi D} \right]^2}$$

$$= \sqrt{(10 \text{ mm})^2 - \left[ 10 \text{ mm} - \frac{(2)(500 \text{ kg})}{(450)(\pi)(10 \text{ mm})} \right]^2} = 1.19 \text{ mm}$$
7.12 Consider a metal single crystal oriented such that the normal to the slip plane and the slip direction are at angles of 43.1° and 47.9°, respectively, with the tensile axis. If the critical resolved shear stress is 20.7 MPa (3000 psi), will an applied stress of 45 MPa (6500 psi) cause the single crystal to yield? If not, what stress will be necessary?

Solution

This problem calls for us to determine whether or not a metal single crystal having a specific orientation and of given critical resolved shear stress will yield. We are given that φ = 43.1°, λ = 47.9°, and that the values of the critical resolved shear stress and applied tensile stress are 20.7 MPa (3000 psi) and 45 MPa (6500 psi), respectively. From Equation 7.2

\[ \tau_R = \sigma \cos \phi \cos \lambda = (45 \text{ MPa})(\cos 43.1°)(\cos 47.9°) = 22.0 \text{ MPa (3181 psi)} \]

Since the resolved shear stress (22 MPa) is greater than the critical resolved shear stress (20.7 MPa), the single crystal will yield.
7.17 Consider a single crystal of some hypothetical metal that has the FCC crystal structure and is oriented such that a tensile stress is applied along a \([\overline{1}02]\) direction. If slip occurs on a \((111)\) plane and in a \([\overline{1}01]\) direction, compute the stress at which the crystal yields if its critical resolved shear stress is 3.42 MPa.

Solution

This problem asks for us to determine the tensile stress at which a FCC metal yields when the stress is applied along a \([\overline{1}02]\) direction such that slip occurs on a \((111)\) plane and in a \([\overline{1}01]\) direction; the critical resolved shear stress for this metal is 3.42 MPa. To solve this problem we use Equation 7.4; however it is first necessary to determine the values of \(\phi\) and \(\lambda\). These determinations are possible using Equation 7.6. Now, \(\lambda\) is the angle between \([\overline{1}02]\) and \([\overline{1}01]\) directions. Therefore, relative to Equation 7.6 let us take \(u_1 = -1, v_1 = 0, w_1 = 2\) as well as \(u_2 = -1, v_2 = 0, w_2 = 1\). This leads to

\[
\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}}\right]
\]

\[
= \cos^{-1}\left\{\frac{(-1)(-1) + (0)(0) + (2)(1)}{\sqrt{[(-1)^2 + (0)^2 + (2)^2][(-1)^2 + (0)^2 + (1)^2]}}\right\}
\]

\[
= \cos^{-1}\left(\frac{3}{\sqrt{10}}\right) = 18.4^\circ
\]

Now for the determination of \(\phi\), the normal to the \((111)\) slip plane is the \([111]\) direction. Again using Equation 7.6, where we now take \(u_1 = -1, v_1 = 0, w_1 = 2\) (for \([\overline{1}02]\)), and \(u_2 = 1, v_2 = 1, w_2 = 1\) (for \([111]\)). Thus,

\[
\phi = \cos^{-1}\left[\frac{(-1)(1) + (0)(1) + (2)(1)}{\sqrt{[(-1)^2 + (0)^2 + (2)^2][1^2 + 1^2 + 1^2]}}\right]
\]

\[
= \cos^{-1}\left(\frac{3}{\sqrt{15}}\right) = 39.2^\circ
\]

It is now possible to compute the yield stress (using Equation 7.4) as
\[
\sigma_y = \frac{\tau_{crs}}{\cos \phi \cos \lambda} = \frac{3.42 \text{ MPa}}{\left(\frac{3}{\sqrt{10}}\right) \left(\frac{3}{\sqrt{15}}\right)} = 4.65 \text{ MPa}
\]